The "Parallax Compression" prime pattern: exact structure, limit theorems, and the odd-width rule

Working note

July 22, 2026

Abstract

In 2018 N. Spivack and S. Gilchrist announced a visualization of the primes ("Parallax Compression", also called the "Daena pattern") in which blocks of N integers are arranged in a pyramid and a cell is marked precisely when it contains a prime. They observed empirically that for even cell widths N the marked pattern approaches the coprimality triangle $T(n,k) = [\gcd(n,k) = 1]$ (OEIS A054521), while for odd N a different recurring pattern appears which they could not explain. We formalize the construction and prove: (i) each cell is an N-term arithmetic progression occupying a full residue class $C^* \bmod R$ of its row window, where $C^* = C + \frac{R}{2}[2 \mid R][2 \nmid N]$ depends on N only through its parity; (ii) cells with $\gcd(R, C^*) > 1$ contain no primes, for *every* width $N \geq 2$; (iii) by the prime number theorem for arithmetic progressions, every other cell is marked for all sufficiently large N of fixed parity, so the pattern converges pointwise to A054521 for even N and, resolving the open question, to an explicit "half-twist" of A054521 for odd N — the same triangle with each row $R \equiv 2 \pmod 4$ rotated by $R/2$; (iv) by Siegel–Walfisz the convergence is uniform in rows $R \leq (\log N)^A$; (v) the pattern produced by the integers coprime to 30030 agrees with an explicit gcd rule *exactly* for all $N \geq 22$, via Jacobsthal's function $g(30030) = 22$. Computational controls show the entire visual structure is sieve structure: a primorial control reproduces it almost perfectly, while a Cramér random control destroys it. Two conjectures supported by exhaustive computation ($2 \leq N \leq 420$ and beyond) are stated, including that the rendered N-row pattern equals its limit *exactly* for every $N \geq 75$.

1 Introduction

In May 2018, Nova Spivack announced [1] a prime-number visualization due to Shaun Gilchrist, called *Parallax Compression* (or the *Daena pattern*), with implementations published by Gilchrist [2, 3]. Integers are packed into a pyramid of cells, each cell covering N integers; a cell is colored black if it contains at least one prime and red otherwise. The announcement reported two empirical findings.

1. For *even* N, as N grows the black/red pattern converges to the triangle of OEIS sequence A054521 [5], $T(n,k) = 1$ iff $\gcd(n,k) = 1$;
2. for *odd* N a *different* pattern appears, which also recurs across different odd values of N; the discoverers stated explicitly that this pattern "is not simply the GCD sequence" and that the variance was not understood.

A contemporaneous Hacker News discussion [4] reverse-engineered the construction, observed that for even N the condition $\gcd(R, C) = 1$ (row R, column C) is necessary for a cell to contain

primes, noted by a Cramér-type heuristic that it should usually be sufficient, and exhibited the counterexample $(R, C) = (13, 3)$ at $N = 14$. No part of this was developed formally, no limit statement was formulated or proved, and the odd-N pattern remained unexplained.

This note supplies the missing mathematics. Our contributions:

- an exact formalization of the transform (Definition 2.1), validated against the discoverers' own code and its known fingerprints;
- a residue lemma (Lemma 3.1) showing each cell is a full residue class of a window, with effective column $C^* = C + \frac{R}{2}\,[2 \mid R]\,[2 \nmid N]$ — in particular the geometry depends on N only through its parity, explaining why one pattern recurs for all even N and another for all odd N;
- the forced-composite theorem (Theorem 3.3): for every $N \geq 2$, $\gcd(R, C^*) > 1$ implies the cell is prime-free — an exact, not asymptotic, statement;
- pointwise convergence (Theorem 4.1) to A054521 for even N, and for odd N to an explicit *half-twist* of A054521 (Corollary 4.2, Proposition 4.3), resolving the discoverers' open question;
- a uniform unconditional convergence strip $R \leq (\log N)^A$ via Siegel–Walfisz (Theorem 4.4);
- an exact characterization of the primorial control pattern for $N \geq 22$ via Jacobsthal's function (Theorem 5.1);
- computational controls separating sieve structure from genuinely prime-specific content (Section 5), and two conjectures with exhaustive numerical evidence (Section 6).

All computations reported here were independently reverified from the definitions alone (including fresh sieves and fresh random seeds); every numerical claim below reproduced exactly.

2 The transform

Throughout, $T(m) = m(m+1)/2$ denotes the m-th triangular number, p always denotes a prime, and $[P]$ is the Iverson bracket.

Definition 2.1 (Parallax compression)**.** Fix a *cell width* $N \geq 1$. For a *row* $R \geq 1$ and *column* C with $1 \leq C \leq R$, define the cell

$$S_N(R, C) \;=\; \big\{\, N\,T(R-1) + C + iR \;:\; i = 0, 1, \ldots, N-1 \,\big\},$$

an N-term arithmetic progression with common difference R. The R cells of row R use each integer of the *window* $W_N(R) = \big(N\,T(R-1),\; N\,T(R)\big]$ exactly once. A cell is *marked* iff $S_N(R, C)$ contains at least one prime. The rows, drawn centered with marked cells black and unmarked cells red, form the published pyramid.

Equivalently, in the discoverers' formulation [2]: row R takes the next NR integers, splits them into N consecutive blocks of length R, and cell C collects the C-th entry of every block. This transposition is what turns cells into arithmetic progressions, and it is the source of all structure below.

Remark 2.2 (Provenance and validation). Definition 2.1 was extracted from the JavaScript of [2] (`layerToTerms/layerToSides/sidesToCol/maskPrimeTest`) and validated two ways: the published description "the leading cell of row 2 contains n numbers between $n+1$ and $3n$ with an increment of 2" matches $S_N(2,1) = \{N+1, N+3, \ldots, 3N-1\}$; and the implementation reproduces the deviation fingerprint reported in [4]: at $N = 14$ the unique coprime prime-free cell in the first 14 rows is exactly $(R, C) = (13, 3)$.

3 The residue lemma and forced composites

Lemma 3.1 (Residue structure). *Let $N, R \geq 1$ and $1 \leq C \leq R$. Set*

$$\varepsilon \;=\; \varepsilon(N,R) \;=\; [\,2 \mid R\,]\,[\,2 \nmid N\,], \qquad C^* \;=\; C + \varepsilon\,\tfrac{R}{2}.$$

Then:

(a) $N\,T(R-1) \equiv \varepsilon\,\frac{R}{2} \pmod R$*; hence every element of* $S_N(R,C)$ *is* $\equiv C^* \pmod R$.

(b) $S_N(R,C) = \{\, x \in W_N(R) : x \equiv C^* \pmod R \,\}$*: the cell consists of* all *integers of its window lying in the residue class* $C^* \bmod R$.

Proof. (a) We compute $N\,T(R-1) = NR(R-1)/2 \bmod R$. If N is even, this is $R \cdot \frac{N}{2}(R-1) \equiv 0$. If N and R are both odd, $\frac{R-1}{2} \in \mathbb{Z}$, so it equals $R \cdot N\frac{R-1}{2} \equiv 0$. If N is odd and R is even, write it as $\frac{R}{2} \cdot N(R-1)$; here $N(R-1)$ is odd, say $N(R-1) = 2k+1$, and $\frac{R}{2}(2k+1) = kR + \frac{R}{2} \equiv \frac{R}{2} \pmod R$. In all cases $N\,T(R-1) \equiv \varepsilon\frac{R}{2} \pmod R$, and each element $N\,T(R-1) + C + iR \equiv \varepsilon\frac{R}{2} + C = C^* \pmod R$.

(b) The elements of $S_N(R,C)$ form an arithmetic progression with step R inside $W_N(R)$: the least is $N\,T(R-1) + C \geq N\,T(R-1) + 1$ and the largest is $N\,T(R-1) + C + (N-1)R = N\,T(R) + C - R \leq N\,T(R)$ (since $C \leq R$). Conversely, the integers of $W_N(R)$ congruent to $N\,T(R-1) + C \pmod R$ form an arithmetic progression with step R whose least element is $N\,T(R-1) + C$ (because $1 \leq C \leq R$) and whose largest is $N\,T(R) + C - R$ (because the next one, $N\,T(R) + C$, exceeds $N\,T(R)$). The two progressions coincide. □

Remark 3.2 (Only the parity of N matters). C^* depends on N only through $N \bmod 2$. This single fact explains the discoverers' observation that one pattern recurs for all even N and a different one for all odd N. All congruence statements below are invariant under reduction of C^* modulo R, so the possible excess $C^* > R$ is immaterial.

Theorem 3.3 (Forced composite cells). *Let $N \geq 2$, $R \geq 2$, $1 \leq C \leq R$, and suppose $d = \gcd(R, C^*) > 1$. Then $S_N(R,C)$ contains no prime. Thus the cell is unmarked for* every *such width N — not only in a limit.*

Proof. By Lemma 3.1(a), every $x \in S_N(R,C)$ satisfies $x \equiv C^* \pmod R$; since $d \mid R$ and $d \mid C^*$, we get $d \mid x$ for every element. It remains to check that every element exceeds d, so that $x = d\,(x/d)$ with $x/d \geq 2$ is composite. The least element is $m := N\,T(R-1) + C$. If $R \geq 3$ then, using $N \geq 1$,

$$m \;\geq\; \tfrac{R(R-1)}{2} + 1 \;=\; \tfrac{R^2}{2} - \tfrac{R}{2} + 1 \;>\; R \;\geq\; d,$$

since $\frac{R^2}{2} - \frac{R}{2} + 1 - R = \frac{(R-1)(R-2)}{2} > 0$ for $R \geq 3$. If $R = 2$ then, using $N \geq 2$, $m = N + C \geq 3 > 2 = R \geq d$. Hence every element is composite. □

Remark 3.4 (Sharpness). The hypothesis $N \geq 2$ cannot be dropped: $(N, R, C) = (1, 2, 1)$ gives $S_1(2, 1) = \{2\}$, which is prime, while $C^* = 2$ and $\gcd(2, 2) = 2$. This is the *only* failure: a prime element divisible by $d > 1$ must equal $d \leq R$, forcing $N\,T(R-1) + C \leq R$ and hence $R \leq 2$, $(N, R, C) = (1, 2, 1)$.

4 Limit theorems

The empirical claim "the pattern converges to A054521" cannot hold uniformly over all rendered rows in a naive sense: for small N, deep rows $R \gg N$ contain many coprime prime-free cells (Section 6). The statements that are true — and provable — are pointwise convergence along each parity class (Theorem 4.1), uniform convergence in a strip of rows growing with N (Theorem 4.4), and, conjecturally, exact agreement on the standard N-row rendering for all $N \geq 75$ (Conjecture 6.1).

Theorem 4.1 (Pointwise limit)**.** *Fix $R \geq 1$ and $1 \leq C \leq R$, fix a parity for N, and let C^* be as in Lemma 3.1 (it depends only on the parity). If $\gcd(R, C^*) = 1$, then for all sufficiently large N of that parity the cell $S_N(R, C)$ contains a prime; moreover the number of primes in $S_N(R, C)$ tends to infinity as $N \to \infty$ along that parity.*

Proof. For $R = 1$, $S_N(1, 1) = \{1, \ldots, N\}$ and the claim is the prime number theorem. Let $R \geq 2$ and put $a = T(R-1) \geq 1$, $b = T(R)$, so by Lemma 3.1(b) the primes in $S_N(R, C)$ are exactly the primes $p \in (aN, bN]$ with $p \equiv C^* \pmod R$, and their number is

$$\pi(bN; R, C^*) - \pi(aN; R, C^*).$$

Since $\gcd(C^*, R) = 1$ and R is *fixed*, the prime number theorem for arithmetic progressions (de la Vallée Poussin; see [7, §20]) gives $\pi(x; R, C^*) = \mathrm{Li}(x)/\varphi(R) + E(x)$ with $E(x) = O_R\Big(xe^{-c\sqrt{\log x}}\Big) = o(x/\log x)$. Hence the count equals

$$\frac{\mathrm{Li}(bN) - \mathrm{Li}(aN)}{\varphi(R)} + o\Big(\frac{N}{\log N}\Big) \qquad (N \to \infty,\ R, a, b \text{ fixed}).$$

Now $\mathrm{Li}(bN) - \mathrm{Li}(aN) = \int_{aN}^{bN} \frac{dt}{\log t} \geq \frac{(b-a)N}{\log(bN)}$ and $b - a = R$, so the main term is at least $\frac{RN}{\varphi(R)\log(bN)} \geq \frac{N}{\log(bN)} \asymp \frac{N}{\log N}$, which dominates the error. The count tends to infinity. □

Corollary 4.2 (Limit patterns; resolution of the odd-N question)**.** *Define*

$$T_{\mathrm{even}}(R, C) = [\gcd(R, C) = 1], \qquad T_{\mathrm{odd}}(R, C) = \big[\gcd\big(R,\ C + \tfrac{R}{2}[2 \mid R]\big) = 1\big].$$

For every fixed cell (R, C): along even N the cell is eventually marked iff $T_{\mathrm{even}}(R, C) = 1$, and along odd N iff $T_{\mathrm{odd}}(R, C) = 1$. In the complementary cases the cell is unmarked for all $N \geq 2$ of the parity (Theorem 3.3). Thus the pattern converges pointwise to the A054521 triangle for even N, and for odd N to the "half-twist" triangle T_{odd}.

Proposition 4.3 (Structure of the half-twist)**.**

(a) If R is odd or $R \equiv 0 \pmod 4$, row R of T_{odd} equals row R of T_{even}.

(b) If $R \equiv 2 \pmod 4$, row R of T_{odd} is row R of T_{even} cyclically rotated by $R/2$, and the two rows differ.

(c) Both triangles have row sums $\varphi(R)$.

Consequently the odd-width limit differs from A054521 precisely on the rows $R \equiv 2 \pmod 4$.

Proof. (a) For odd R the shift vanishes. Let $4 \mid R$ and let $p \mid R$ be prime. If p is odd then $v_p(R/2) = v_p(R) \geq 1$, so $p \mid \frac{R}{2}$ and $C + \frac{R}{2} \equiv C \pmod p$. If $p = 2$ then $v_2(R/2) = v_2(R) - 1 \geq 1$, so $\frac{R}{2}$ is even and again $C + \frac{R}{2} \equiv C \pmod 2$. Since $\gcd(R, y) = 1$ iff no prime divisor of R divides y, and $C + \frac{R}{2} \equiv C$ modulo every prime divisor of R, coprimality is preserved both ways.

(b) For even R, $C \mapsto C + \frac{R}{2} \pmod R$ is a translation of $\mathbb{Z}/R\mathbb{Z}$, so the odd row is a cyclic rotation of the even row by $R/2$. If $R \equiv 2 \pmod 4$ then $\frac{R}{2}$ is odd, so C and $C + \frac{R}{2}$ have opposite parities; every C coprime to the even number R is odd, hence $C + \frac{R}{2}$ is even and $\gcd(R, C + \frac{R}{2}) \geq 2$. The marked sets of the two rows are therefore disjoint, and both are nonempty ($\varphi(R) \geq 1$), so the rows differ.

(c) A translation of $\mathbb{Z}/R\mathbb{Z}$ permutes residue classes and so preserves the number of coprime classes, $\varphi(R)$. □

Theorem 4.4 (Uniform strip)**.** *For every $A > 0$ there is N_A such that for all $N \geq N_A$ (either parity), every cell (R, C) with $2 \leq R \leq (\log N)^A$ and $\gcd(R, C^*) = 1$ contains a prime. Consequently, for $N \geq N_A$ the marked/unmarked pattern in rows $R \leq (\log N)^A$ agrees exactly with the parity-appropriate limit triangle of Corollary 4.2.*

Proof. Write $q = R$, $a = T(R-1)$, $b = T(R)$, $x_1 = aN$, $x_2 = bN$. By Lemma 3.1(b) it suffices to produce a prime $p \equiv C^* \pmod q$ with $x_1 < p \leq x_2$.

By the Siegel–Walfisz theorem [7, §22], for every $A > 0$ there are $C_A, c_A > 0$ such that

$$\psi(x; q, a^*) = \frac{x}{\varphi(q)} + O_A\Big(x\, e^{-c_A\sqrt{\log x}}\Big)$$

uniformly over $q \leq (\log x)^A$ and $\gcd(a^*, q) = 1$. Since $x_1, x_2 \geq N$ and $q \leq (\log N)^A \leq (\log x_i)^A$, this applies at both endpoints:

$$\psi(x_2; q, C^*) - \psi(x_1; q, C^*) = \frac{(b-a)N}{\varphi(q)} + O_A\Big(x_2\, e^{-c_A\sqrt{\log N}}\Big).$$

The main term is $\frac{RN}{\varphi(R)} \geq N$. The error is at most $C_A\, T(R)\, N\, e^{-c_A\sqrt{\log N}} \leq C_A(\log N)^{2A} N e^{-c_A\sqrt{\log N}} = o(N)$ uniformly over $R \leq (\log N)^A$.

Next we discard prime powers. With $\theta(x; q, a^*) = \sum_{p \leq x,\, p \equiv a^*} \log p$,

$$0 \leq \psi(x; q, a^*) - \theta(x; q, a^*) \leq \psi(x) - \theta(x) = \sum_{k \geq 2} \theta(x^{1/k}) \leq \frac{\log x}{\log 2}\, \psi(\sqrt{x}) \leq c\sqrt{x} \log x,$$

using Chebyshev's bound $\psi(y) \ll y$ and $\theta(x^{1/k}) = 0$ for $k > \log_2 x$. At $x = x_2 \leq (\log N)^{2A} N$ we have $\sqrt{x} \leq (\log N)^A \sqrt{N}$ and $\log x \leq 2 \log N$ for large N, so this is $O(\sqrt{N}\,(\log N)^{A+1}) = o(N)$.

Combining, $\theta(x_2; q, C^*) - \theta(x_1; q, C^*) \geq N - o(N) > 0$ for all $N \geq N_A$, uniformly in the allowed range. A positive θ-difference exhibits a prime $p \in (x_1, x_2]$ with $p \equiv C^* \pmod R$, which lies in $S_N(R, C)$. □

Remark 4.5. N_A is ineffective (Siegel zeros). Under GRH the same argument with $\psi(x; q, a^*) = x/\varphi(q) + O(\sqrt{x} \log^2 x)$ widens the strip to $R \leq N^{1/2-\delta}$: the error is $O(R\sqrt{N} \log^2(R^2 N))$ against a main term $\geq N$. The full rendered triangle $R \leq N$ would require primes in arithmetic progressions to moduli $q \approx x^{1/3}$ inside windows of length $\approx q\, N$, beyond what is provable even on GRH; this is why Conjecture 6.1 below remains a conjecture.

5 What is prime-specific? Controls

Two control experiments replace the primes by other sets in Definition 2.1. Throughout, $P = 30030 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$ (the 6th primorial).

5.1 The primorial control, exactly

Theorem 5.1 (Primorial control)**.** *Replace "contains a prime" by "contains an integer coprime to P". Let $N \geq 22$, $R \geq 1$, $1 \leq C \leq R$. Then the cell is marked if and only if no prime $p \leq 13$ divides* $\gcd(R, C^*)$.

Proof. If some $p \leq 13$ divides $\gcd(R, C^*)$, then by Lemma 3.1(a) every element is $\equiv C^* \equiv 0 \pmod{p}$, so no element is coprime to P.

Conversely, suppose no $p \leq 13$ divides $\gcd(R, C^*)$. Write the elements as $x_i = x_0 + iR$, $0 \leq i \leq N-1$, with $x_0 = N\,T(R-1) + C$. For $p \leq 13$ with $p \mid R$: every $x_i \equiv C^* \pmod{p}$ and $p \nmid C^*$, so no element is divisible by p; such p impose no constraint. For $p \leq 13$ with $p \nmid R$: $p \mid x_i$ exactly when $i \equiv a_p := -x_0 R^{-1} \pmod{p}$, a single class. Let $m = \prod\{p \leq 13 : p \nmid R\}$ and, by the Chinese remainder theorem, choose s with $s \equiv a_p \pmod{p}$ for all $p \mid m$. Then

$$\gcd(x_i, P) = 1 \iff \forall\, p \mid m :\ i \not\equiv a_p \pmod{p} \iff \gcd(i - s,\, m) = 1.$$

As i runs over $N \geq 22$ consecutive integers, so does $u = i - s$. The maximal gap between consecutive integers coprime to 30030 is 22; this is the value $g(30030) = 22$ of Jacobsthal's function [8, 9] at the 6th primorial (OEIS A048670 [6]; we also verified it by direct scan of a full period). Integers coprime to the divisor m of 30030 are a superset of those coprime to 30030, so their gaps are no larger, and any 22 consecutive integers contain some u with $\gcd(u, m) = 1$. The corresponding x_i is coprime to P. □

The threshold is nearly sharp: the characterization fails for various $N \leq 17$ (e.g. at $N = 17$, cell $(79, 39)$) and was verified to hold with no exceptions for $N \in \{18, \ldots, 21\}$ in rows ≤ 150 and for $N \in \{22, 23, 30, 45, 74, 105\}$ in rows ≤ 120.

5.2 Controls at $N = 74$

At the discoverers' default $N = 74$ (74 rows, $T(74) = 2775$ cells) we compared three membership sets: the primes; the integers coprime to P (control (a), a pure sieve set); and a Cramér random set containing each $x \geq 3$ independently with probability $1/\log x$ (control (b), seed 12345; the qualitative outcome is stable across seeds).

	primes	(a) coprime to 30030	(b) Cramér random
unmarked cells	1080	1050	2
agreement with A054521 limit	2774/2775	2746/2775	1696/2775
agreement with prime pattern	—	2745/2775	1695/2775
forced cells marked (of 1079)	0	0	1078
sporadic prime-free coprime cells	1: $(73, 43)$	0	1: $(66, 1)$

The primorial control is visually indistinguishable from the prime pattern: all glyphs, nested triangles, and row banding are present. Its 29 deviations from the limit are *exactly* the cells with

$\gcd(R, C) > 1$ all of whose prime factors exceed 13 (rows and columns sharing a prime factor ≥ 17, e.g. $(17, 17)$, $(34, 17)$, $(51, 34)$, $(74, 37)$), as Theorem 5.1 predicts. The Cramér control destroys the pattern entirely: 1078 of the 1079 forced-composite cells become marked and the rendering is a nearly solid black pyramid.

Verdict. Every visual feature of the Daena pattern — the pyramid glyphs, nested triangles, banding, and the even/odd dichotomy — survives the sieve control and dies under the random control. The pattern is *sieve structure*: congruence information from small primes. The only genuinely prime-specific content is (i) the sporadic prime-free coprime cells — prime-gap events occurring at roughly the frequency a random model predicts, though concentrated on prime rows (Conjecture 6.2), reflecting the $R/\varphi(R)$ density boost absent from the naive random model — and (ii) the unmarked cells whose gcd has only prime factors > 13, which any fixed primorial control eventually misses.

6 Conjectures and numerical evidence

Call a cell (R, C) with $R \leq N$ an *exception* (at width N) if $\gcd(R, C^*) = 1$ but $S_N(R, C)$ is prime-free — i.e. the rendered pattern deviates there from its limit triangle. An exhaustive scan of all $2 \leq N \leq 420$ and spot checks $N \in \{450, 500, 550, 600\}$ (windows up to 1.08×10^8) found exactly 21 exceptions, listed as $(N; R, C)$:

$(7; 7, 1)$	$(11; 11, 7)$	$(13; 13, 2)$	$(14; 13, 3)$	$(15; 13, 12)$	$(20; 17, 8)$	$(20; 19, 1)$
$(21; 13, 10)$	$(21; 17, 4)$	$(23; 13, 6)$	$(23; 19, 9)$	$(25; 23, 7)$	$(29; 19, 1)$	$(30; 17, 7)$
$(30; 19, 1)$	$(38; 37, 8)$	$(44; 31, 2)$	$(44; 37, 2)$	$(47; 31, 11)$	$(50; 43, 13)$	$(74; 73, 43)$

Conjecture 6.1 (Exactness of the rendered pattern)**.** *For every $N \geq 75$, the pattern in rows $1 \leq R \leq N$ equals the parity-appropriate limit triangle exactly: a cell is unmarked iff $\gcd(R, C^*) > 1$.*

Conjecture 6.2 (Exceptions live on prime rows)**.** *Every exception with $N \geq 2$ has R prime.*

Conjecture 6.1 is verified for $75 \leq N \leq 420$ and $N \in \{450, 500, 550, 600\}$ (zero exceptions); Conjecture 6.2 holds for all 21 known exceptions (rows $7, 11, 13, 17, 19, 23, 31, 37, 43, 73$, all prime).

Heuristic support (not rigorous; assumes Cramér-type independence). A coprime cell in row R is an N-term progression at height $\asymp NT(R)$ whose terms lie in reduced classes, where primes have density $\frac{R}{\varphi(R)} \cdot \frac{1}{\log(NT(R))}$; modeling terms as independent,

$$\Pr[\text{cell prime-free}] \approx \exp\left(-\frac{N\, R/\varphi(R)}{\log(N\, T(R))}\right).$$

Summing over the $\sum_{R \leq N} \varphi(R) \sim \frac{3}{\pi^2} N^2$ coprime cells bounds the expected number of exceptions by roughly $N^2 \exp(-N/(3 \log N))$, which is summable in N; a Borel–Cantelli heuristic then predicts a finite total list of exceptions, consistent with the data (last exception at $N = 74$). For Conjecture 6.2: composite R have $R/\varphi(R) \geq \frac{3}{2}$, which at least squares the prime-free probability relative to prime rows ($R/\varphi(R) = 1 + \frac{1}{R-1}$), so rare exceptions concentrate on prime rows. We emphasize both arguments are heuristics, not proofs.

Beyond the rendered window. The restriction $R \leq N$ matters. For fixed small N the deep rows $R \gg N$ are exception-riddled (at $N = 6$ there are tens of thousands of exceptions by

row 800, including on composite rows, e.g. $(8, 1) = \{57, 65\}$ at $N = 2$), in accordance with the heuristic above as $N/\log(NT(R))$ decays. The empirical statement "the pattern holds for N rows" marks the regime where exceptions have not yet appeared; Theorems 4.1 and 4.4 and Conjecture 6.1 are its correct formalizations at three levels of strength.

7 Prior art and discussion

The even-N forced direction (for even N, $\gcd(R, C) > 1$ implies a prime-free cell) and the insufficiency of coprimality were observed informally in the 2018 Hacker News thread [4], including a Cramér-type heuristic and the $(14; 13, 3)$ counterexample; an A054521 overlay was also posted there. None of this was formalized, and the odd-N pattern was left unexplained both there and in [1]. The OEIS entry for A054521 [5] contains no reference to primes-in-cells, parallax compression, or the present connection. To our knowledge, the residue lemma with its parity-only dependence, the half-twist description of the odd-width limit (Corollary 4.2, Proposition 4.3), the formal convergence statements (Theorems 4.1 and 4.4), and the Jacobsthal-based exact control characterization (Theorem 5.1) are new.

The construction belongs to a visual genre going back to Klauber's 1932 prime triangle and the Ulam spiral [10, 11], where diagonal features reflect prime-rich quadratics. The mechanism here is different and, unusually for this genre, completely analyzable: the transpose-into-progressions step converts the picture into a table of congruence conditions (rows = moduli, cells = residue classes), and the picture converges to the reduced-residue indicator table. The natural next step is an unconditional *density* version of Conjecture 6.1 — that the number of cells in rows $R \le N$ disagreeing with the limit is $o(N^2)$ — which each row being a full residue decomposition of a short window makes a natural target for short-interval Barban–Davenport–Halberstam-type mean value theorems.

Acknowledgments

The construction is due to Shaun Gilchrist; the pattern was publicized by Nova Spivack. All results were verified computationally, and the full package (code, images, and an independent adversarial reverification from the definitions alone) accompanies this note.

References

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